

Optical conductivity in the normal state fullerene superconductors

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We calculate the optical conductivity, $\sigma(\omega)$, in the normal state fullerene superconductors by self-consistently including the impurity scatterings, the electron-phonon and electron-electron Coulomb interactions. The finite bandwidth of the fullerenes is explicitly considered, and the vertex corection is included *a la* Nambu in calculating the renormalized Green's function. $\sigma(\omega)$ is obtained by calculating the current-current correlation function with the renormalized Green's function in the Matsubara frequency and then performing analytic continuation to the real frequency at finite temperature. The Drude weight in $\sigma(\omega)$ is strongly suppressed due to the interactions and transferred to the mid-infrared region around and above 0.06 eV which is somewhat less pronounced and much broader compared with the experimental observation by DeGiorgi *et al.*

I. INTRODUCTION

The optical spectra of fullerene superconductors in the normal state were found to exhibit some unusual features [1,2]. The optical conductivity, $\sigma(\omega)$, deviates considerably from the simple Drude behavior expected for conventional metals: the spectral weight of the Drude peak is reduced by about an order of magnitude and transferred to a mid-infrared (MIR) region around 0.06 eV. This suggests that the strong interaction effects due to the Coulomb and electron-phonon interactions should be important in the optical spectra of the fullerenes. Understanding this unusual behavior in the optical conductivity, therefore, could reveal important information about the fullerenes and contribute to understanding other physical properties of the material.

The optical conductivity $\sigma(\omega)$ represents the rate at which electrons absorb the incident photons at energy ω , and is a useful probe in determining electronic characteristics of the material under study. For an ideal free electron gas, where the interactions between the electrons, and between the electrons and phonons are neglected, and the impurity scattering rate $1/\tau \rightarrow 0$, $\sigma(\omega)$ collapses to a delta-function, $\sigma(\omega) = D_{tot}\delta(\omega)$, where the coefficient D_{tot} represents the total spectral weight. In this case, the optical conductivity sum rule, $\int_0^\infty d\omega \sigma(\omega) = D_{tot} = \pi e^2 n/m$, where n is the density and m is the mass of the electrons, is exhausted entirely by the delta function Drude contribution alone. When the material becomes dirtier, the Drude peak of the optical conductivity acquires the Lorentzian shape with the width of $1/\tau$. The conductivity sum rule is still exhausted by the Drude part alone when only the impurity scatterings are present in the system. The total weight D_{tot} , however, can change as the impurity scatterings or other interactions are introduced when we consider a finite bandwidth, because the projection to a restricted basis set disregards all excitations to higher energy than the bandwidth. When other interactions are present, the free-carrier Drude weight is reduced by the quasiparti-

cle renormalization factor Z such that $D = Z^{-1}D_{tot}$, and the missing spectral weight from the Drude part is transferred to a higher energy region of $\sigma(\omega)$ reflecting the excitation of incoherent scatterings.

The experimentally measured $\sigma(\omega)$ in the normal state A_3C_{60} shows a remarkable reduction of Drude weight and, concomitantly, a pronounced MIR absorption below the inter-band absorption peak: DeGiorgi *et al.* found a pronounced MIR peak around 0.06 eV and analyzed that the Drude weight is reduced to about 0.1 – 0.2 of the total intra-band spectral weight [1], while Iwasa *et al.* observed the MIR absorption peak around 0.4 eV and determined that the Drude weight is reduced to about 0.6 of the total intra-band spectral weight [2]. Although their results show somewhat different Drude weight and MIR absorption energy each other, the pronounced suppression of the Drude weight and the accompanying MIR absorption imply strong electron-phonon and/or electron-electron interactions in this material.

In order to understand this unusual feature in $\sigma(\omega)$ of doped fullerenes, Gunnarsson *et al.* studied the effects of the electron-phonon interaction on $\sigma(\omega)$ assuming that the Migdal theorem is valid [3]. They showed that the electron-phonon interaction leads to a narrowing of the Drude peak by the factor $Z = 1 + \lambda$, where λ is the dimensionless electron-phonon coupling constant, and a transfer of the depleted Drude weight to a MIR region at somewhat larger energies than the phonon energy. Their results, however, are far from sufficient to describe experimental observations. Therefore, they hinted that the Coulomb interaction between conduction electrons, which is neglected in their study, could lead to further reduction of the Drude weight and more pronounced MIR absorption. On the other hand, one of the present authors recently found, by studying the NMR coherence peak suppression in the fullerene superconductors, that the Coulomb interaction between conduction electrons, characterized by $UN_F \approx 0.3 - 0.4$, where U is the effective Coulomb interaction and N_F is the density of states (DOS) at the Fermi level, should be included in addition

to the electron-phonon interaction to understand the various experimental observations in fullerenes in a coherent way [4]. We, therefore, included the electron-electron as well as electron-phonon interactions at the presence of the impurity scatterings in the present paper, to better understand the experimentally observed unusual features in the optical spectra of the fullerene superconductors in the normal state.

For fullerene superconductors, the Fermi energy $\varepsilon_F = B/2 \approx 0.2 - 0.3$ eV and the average phonon frequency $\omega_{ph} \approx 0.05 - 0.15$ eV, where B is the bandwidth. Therefore, $\omega_{ph}/\varepsilon_F \sim 1$ for fullerenes unlike conventional metals, where $\omega_{ph}/\varepsilon_F \ll 1$. When $\omega_{ph}/\varepsilon_F \sim 1$, the phonon vertex correction becomes important because the Migdal theorem does not hold [5–7], and the frequency dependence of the effective Coulomb interaction, $V_{eff}(\omega)$, should be considered because the frequency scale at which $V_{eff}(\omega)$ varies is comparable with that of electron-phonon interaction [8]. In this present work, concerned with the effects of the Coulomb and electron-phonon interactions on the optical spectra in the narrow band fullerene superconductors, the vertex correction is incorporated in calculating the electron self-energy [4,9,10]. The Coulomb interaction, modelled in terms of the onsite Hubbard repulsion, is included on an equal footing with the electron-phonon interaction, and considered fully self-consistently in calculating the effective electron-electron interaction [4,10]. The effective electron-electron interaction becomes frequency dependent through the screening. The impurity effects are included with the t -matrix approximation.

Through the relation

$$\Sigma(ip) = G^{-1}(ip) - G_0^{-1}(ip), \quad (1)$$

one obtains the electron self-energy $\Sigma(ip)$ in the Mastubara frequency, which gives $\Sigma(\omega)$ in the real frequency after the analytic continuation. G_0 and G are, respectively, the bare and renormalized electron Green's functions. $\Sigma(\omega)$ or $Z(\omega)$, where the renormalization function $Z(\omega)$ is given by $\Sigma(\omega) = \omega - \omega Z(\omega)$, defines the single-particle Green's function of an interacting system as

$$G^{-1} = \omega - \xi_k - \Sigma(\omega) = \omega Z(\omega) - \xi_k, \quad (2)$$

where ξ_k is the electron energy measured from the chemical potential, $\xi_k = \varepsilon_k - \mu$. Then, the optical conductivity can be obtained by calculating the current-current correlation function, $\Pi(i\omega)$, using the renormalized Green's function obtained from solving Eq. (1) self-consistently. The calculated optical conductivity shows a strong reduction of Drude weight and a broad MIR absorption, although the MIR feature around 0.06 eV is less pronounced and broader compared with experimental observations.

This paper is organized as follows: In the following section, we present the Eliashberg-type formalism in the Matsubara frequency to calculate the renormalized Green's function with the impurity, electron-phonon,

and Coulomb interactions included self-consistently. We then describe the analytic continuation procedure to obtain the renormalization function $Z(\omega)$ in the real frequency. The optical conductivity calculated with the renormalized Green's function is presented in Sec. III. We will discuss how the Drude part and the MIR absorption of $\sigma(\omega)$ are affected as the impurity scattering rate, the electron-phonon and electron-electron interactions are varied. These results will then be compared with the experimental observations. Finally, Sec. IV is for the summary and some concluding remarks.

II. FORMALISM

The optical conductivity is calculated from the current-current correlation function, $\Pi(\omega)$, as $\sigma(\omega) = \frac{i}{\omega} \lim_{q \rightarrow 0} \Pi(q, \omega)$ [11]. We use the approximation where the electron self energy is momentum independent. In this case, it can be shown that the vertex correction in the current-current correlation function vanishes for $q \rightarrow 0$ [12]. This leads to

$$\Pi(i\omega_m) = \frac{2e^2}{3m^2V} \sum_{\vec{p}} \vec{p}^2 \frac{1}{\beta} \sum_{ip_n} G(\vec{p}, ip_n + i\omega_m) G(\vec{p}, ip_n), \quad (3)$$

where $ip_n = \pi T(2n + 1)$ and $i\omega_m = 2\pi Tm$ are, respectively, fermion and boson Matsubara frequencies, where T is the temperature, m and n are the integers. $\beta = 1/k_B T$, and V is the volume. The evaluation of Eq. (3) using Eq. (2) produces

$$\Pi(i\omega) = \frac{2\pi e^2 n}{m} \frac{1}{\beta} \sum_{ip} \frac{\theta(ip + i\omega) - \theta(ip)}{(p + \omega)Z(ip + i\omega) - pZ(ip)} \quad (4)$$

in the Matsubara frequency. After performing the analytic continuation of $i\omega \rightarrow \omega + i\delta$, to the real frequency, the optical conductivity is given by

$$\begin{aligned} \sigma(\omega) = & \frac{1}{\omega} \frac{e^2 n}{m} \int_{-\infty}^{\infty} d\varepsilon [f_F(\varepsilon) - f_F(\varepsilon + \omega)] \\ & \times \text{Re} \left[i \frac{\theta(\varepsilon + i\delta) - \theta(\varepsilon + \omega + i\delta)}{\varepsilon Z(\varepsilon + i\delta) - (\varepsilon + \omega)Z(\varepsilon + \omega + i\delta)} \right. \\ & \left. - i \frac{\theta(\varepsilon - i\delta) - \theta(\varepsilon + \omega - i\delta)}{\varepsilon Z(\varepsilon - i\delta) - (\varepsilon + \omega)Z(\varepsilon + \omega - i\delta)} \right], \end{aligned}$$

where $f_F(\varepsilon) = 1/(1 + e^{\beta\varepsilon})$ is the Fermi distribution function, and $\theta(\omega + i\delta) = \tan^{-1} [\frac{i\varepsilon_F}{\omega Z(\omega + i\delta)}]$. The finite conduction bandwidth B with a constant DOS is explicitly considered through the factor of θ , which is $\pi/2$ for the usual case of infinite bandwidth metal. In order to calculate the optical conductivity from Eq. (5) we need $Z(\omega)$ which defines single-particle interacting Green's function

$G(\omega)$. This can be obtained by solving Eq. (1) self-consistently. The electron self-energy is obtained by calculating the exchange diagram of the renormalized electron Green's function and the effective electron-phonon and Coulomb interactions with the vertex correction included via the method of Nambu. The Coulomb interaction, modelled in terms of the onsite Hubbard repulsion for simplicity, is included on an equal footing with the electron-phonon interaction. The impurity effects are included with the t -matrix approximation. The Eliashberg-type equation can be written in the Mastubara frequency as

$$Z_n p_n = p_n + \frac{1}{\beta} \sum_m [\lambda_{ph}(n-m) - \lambda_{ch}(n-m) + \lambda_{sp}(n-m)] 2\theta_m \Gamma + \frac{1}{\pi\tau} \theta_n, \quad (6)$$

where $\theta_n = \tan^{-1}(\frac{B}{2p_n Z_n})$, and $\lambda_{ph}(n-m) = \int_0^\infty d\Omega \frac{\alpha^2 F(\Omega) 2\Omega}{[\Omega^2 + (p_n - p_m)^2]}$ is the electron-phonon interaction kernel. $\lambda_{ch}(n-m)$ and $\lambda_{sp}(n-m)$ are, respectively, the interactions in the charge and spin channels due to the Hubbard repulsion. They are determined self-consistently as

$$\lambda_{ch}(k) = U N_F \left\{ \frac{1}{2} - \chi_n + \chi_n^2 \ln[1 + 1/\chi_n] \right\}, \quad (7)$$

$$\lambda_{sp}(k) = U N_F \left\{ \frac{1}{2} + \chi_n + \chi_n^2 \ln[1 - 1/\chi_n] \right\}, \quad (8)$$

where $\chi_n(k)$ is the dimensionless susceptibility given by

$$\chi_n(k) = \frac{N_F U}{\varepsilon_F} \frac{1}{\beta} \sum_l \theta_l \theta_{l+k}. \quad (9)$$

The Γ on the right hand side of Eq. (6) represents the vertex correction satisfying the Ward-identity [13]. When we neglect the vertex correction, $\Gamma = 1$. If we assume a weak frequency dependence of Γ , the vertex function Γ reduces to $Z(ip_m)$. In this work, we treat the vertex correction exactly, and Γ is given by

$$\Gamma = \left[\frac{ip_n Z(ip_n) - ip_m Z(ip_m)}{ip_n - ip_m} \right]. \quad (10)$$

Solving Eq. (6) self-consistently yields $Z(i\omega)$ in the Mastubara frequency. In order to calculate $\sigma(\omega)$, analytic continuation of $i\omega \rightarrow \omega + i\delta$ should be performed to get $Z(\omega)$ in real frequency. The numerically exact analytic continuation of standard Eliashberg equation is usually performed by the iterative method developed by Marsiglio, Shossmann, and Carbotte (MSC) using a mixed-representation [14]. But when we include the vertex function exactly, the MSC method can not be applied because it needs a specific form of equation. Here, in order to consider vertex correction exactly, we do the alanytic continuation by employing the iterative method extended

by Takada [15]. In this case, Eq. (6) is transformed to a mixed representation as follow:

$$Z(\omega) = \tilde{Z}(\omega) + \int_0^\infty d\Omega P(\Omega) \left\{ [n_B(\Omega) + n_F(\omega + \Omega)] \times G(\omega + \Omega) \left[\frac{(\omega + \Omega)Z(\omega + \Omega) - \omega Z(\omega)}{\Omega} \right] + [n_B(\Omega) + n_F(\Omega - \omega)] G(\omega - \Omega) \times \left[\frac{(\omega - \Omega)Z(\omega - \Omega) - \omega Z(\omega)}{-\Omega} \right] \right\}, \quad (11)$$

where

$$\begin{aligned} \tilde{Z}(\omega) &= 1 + \frac{1}{\omega\beta} \sum_m \int_0^\infty d\Omega P(\Omega) \left(\frac{1}{ip_m - \omega - \Omega} - \frac{1}{ip_m - \omega + \Omega} \right) \\ &\times G(ip_m) \left[\frac{ip_m Z(ip_m) - \omega Z(\omega)}{ip_m - \omega} \right] + \frac{i}{\pi\tau} \frac{\theta(\omega)}{\omega}, \\ P(\Omega) &= -\frac{1}{\pi} \text{Im} \Lambda(\Omega) \\ \Lambda(\Omega) &= \lambda_{ch}(\Omega) - \lambda_{ph}(\Omega) - \lambda_{sp}(\Omega) \\ G(ip_m) &= 2\theta(ip_m), \quad G^R(\omega) = 2i\theta(\omega). \end{aligned} \quad (12)$$

$\tilde{Z}(\omega)$ of Eq. (11) represents the renormalization function obtained by substituting $i\omega$ to $\omega + i\delta$ *before* the frequency summation. The second term is the correction to $Z(\omega)$ to yield the correct retarded renormalization function $Z(\omega)$ one would have obtained if the analytic continuation were performed *after* the frequency summation. Putting the solutions of Eq. (6), $Z(i\omega)$, into the $\tilde{Z}(i\omega)$ Eq. (11) yields a self-consistent Eliashberg-type equation in the real frequency. Then, $Z(\omega)$ can be obtained by computing iteratively Eq. (11).

In order to model fullerene superconductors, three truncated-Lorentization functions were used to represent $\alpha^2 F(\Omega)$ as follow [4,10]:

$$\begin{aligned} \alpha^2 F(\Omega) &= \sum_{\nu=1}^3 \alpha_\nu^2 F_\nu(\Omega), \\ F_\nu(\Omega) &= \begin{cases} \frac{1}{R} \left[\frac{1}{(\Omega - \omega_\nu)^2 + \Gamma^2} - \frac{1}{\Gamma_c^2 + \Gamma^2} \right], & \text{for } |\Omega - \omega_\nu| \leq \Gamma_c, \\ 0, & \text{otherwise,} \end{cases} \end{aligned} \quad (13)$$

where $F_\nu(\Omega)$ is the truncated Lorentzian centered at ω_ν with the width of $\Gamma = \omega_\nu/5$, Γ_c is the cutoff frequency of $\Gamma_c = 3\Gamma$, and R is normalization constant such that $\int_0^\infty d\Omega F_\nu(\Omega) = 1$. Various theoretical and experimental estimates do not agree well each other in terms of distribution of coupling strength α_ν^2 among different modes. These estimates show, however, that the phonon frequency derived from intramolecular A_g and H_g modes are distributed over 0.03 – 0.2 eV with the total λ in the range of 0.5 – 1 eV. In view of this, we represent the phonon modes with three groups centered around $\omega_\nu = 0.04, 0.09, 0.19$ eV, and $2N_F \alpha_\nu^2 / \omega_\nu = 0.3\lambda_s, 0.2\lambda_s, 0.5\lambda_s$, respectively, for $\nu = 1, 2, 3$. Note that $\sum_{\nu=1}^3 2N_F \alpha_\nu^2 / \omega_\nu = \lambda_s$. The λ_s sets the strength

of $\alpha^2 F(\Omega)$ and $N_F \alpha^2 F(\Omega)/\lambda_s$ is independent of λ_s . For infinite bandwidth superconductors, λ is equal to λ_s in the limit $\Gamma \rightarrow 0$. For a finite bandwidth system, however, λ is reduced from λ_s because the available states to and from which quasiparticles can be scattered are restricted as the bandwidth is reduced.

III. RESULTS

The self-consistent equation of Eq. (1) is solved numerically as described in the previous section to obtain $Z(\omega)$. Then, the optical conductivity is calculated from the Eq. (5). Fig. 1 shows the optical conductivity $\sigma(\omega)$ as λ is varied when the Coulomb interaction U is set to 0 for a reference. Here, the Fermi energy ε_F , temperature T and impurity scattering rate $1/\tau$ are set to 0.25, 0.001, 0.01 eV, respectively. This result shows quite a similar behavior to van den Brink *et al.* calculation. As λ is increased, the width of Drude peak becomes narrower and its weight is transferred to a mid-infrared spectrum. However, the reduction of Drude weight is less than the factor of $(1+\lambda)$, because of the finite bandwidth. The inset shows a MIR absorption spectra obtained by extracting Drude part from the total optical conductivity. In determining the Drude weight, fitting procedure was carefully employed and confirmed by examining zero frequency extrapolation in the Mastubara frequency which is proposed by Scalapino *et al* [16]. The three Lorentzian peaks of $\alpha^2 F(\omega)$ in the electron-phonon pairing kernel are attributed to the development of these MIR peaks. But, the MIR peaks are broadened and move to slightly high frequencies. Fig. 2 shows the MIR absorption due to the Coulomb interaction. The MIR part is also extracted by fitting as shown in the inset. In order to focus on how U affects the total optical conductivity, λ is set to 0. ε_F , T and $1/\tau$ are same as in Fig. 1. The Coulomb interaction induces the strong ω dependence of renormalization function $Z(\omega)$, and the low frequency strong ω dependence of $Z(\omega)$ distorts the Drude part of optical conductivity and induces the MIR absorption in the fairly low frequency region. As the impurity effect is enhanced, the MIR absorption due to Coulomb interactions tends to shift to higher frequency and finally merge together with the MIR peaks developed by electron-phonon interaction, as shown in Fig. 3 for $UN_F = 0.3$ and $\lambda = 0.7$. Note that the position of this merged MIR peak in Fig. 3 is around and above 0.06 eV which is experimentally observed value of DeGiorgi.

Fig. 4 is $\sigma(\omega)$ of doped fullerenes with $T = 0.005$ eV, $\varepsilon_F = 0.25$ eV, $1/\tau = 0.1$ eV, $UN_F = 0.3$, and $\lambda = 0.7$, which is to be compared with the experimental observations. The Drude weight is reduced to 0.467 of the total intra-band optical weight. The reduction factor of the Drude weight by electron phonon interaction is $1 + \lambda$, and the finite bandwidth further restricts the reduction factor. It therefore seems unlikely that the Drude weight

less than about 0.6 of the total intra-band spectral weight can be explained without the Coulomb interactions, when we take $\lambda \approx 0.7 - 0.8$. The Coulomb interaction suppresses the Drude part substantially by inducing ω dependence of the renormalization function $Z(\omega)$ in the low frequency region. We think that the large reduction of Drude weight like the DeGiorgi experiment is a result of the strong Coulomb interaction between conduction electrons in addition to the electron-phonon interaction. However, our results are still not sufficient to explain experimentally founded results: (a) The Drude weight is about 0.46 of the total intra-band optical weight with a reasonable set of parameter values while DeGiorgi found 0.1 – 0.2. (b) The MIR absorption is very broad which begins around 0.02 eV, has a peak around 0.07 eV and extends well over the Fermi energy.

IV. SUMMARY AND CONCLUSION

In this paper, we tried to give an explanation for the unusual behavior of optical conductivity in the normal state A_3C_{60} . It is generally accepted that the fullerene superconductor could be characterized by the phonon-mediated s -wave superconductor [1,8]. However, a few experiments like optical conductivity still remain not understood by the electron-phonon scattering together with the disorder effects. Our motivation lies in that the fullerene superconductors have such a narrow bandwidth that the phonon frequency, the Coulomb interaction, and the Fermi energy are all comparable, $\omega_{ph} \sim V \sim \varepsilon_F$. In order to consider properly the Coulomb interaction and the narrow bandwidth of fullerene superconductors, the self-consistent Eliashberg-type coupled equations are solved to obtain the renormalized Green's function. The theory includes the frequency dependent screened Coulomb interaction together with the electron-phonon interaction and includes the vertex correction via Nambu's method. In order to treat the vertex function exactly, analytic continuation is performed via the iterative method of mixed representation which is developed by Takada. Once we get renormalization function $Z(\omega)$ in real frequency, we can calculate optical conductivity in normal states. As we expected, the electron-phonon interaction is not sufficient to resolve the substantial reduction of Drude weight and pronounced MIR peak. The strong Coulomb interaction induces ω dependence in renormalization function $Z(\omega)$. As a result, the Drude form in optical spectra is distorted accompanying the reduction of Drude weight. When the impurity effect is enhanced, the MIR absorption induced by strong Coulomb interaction merge together with the MIR peaks due to electron-phonon scattering showing large reduction of Drude weight and MIR peak around 0.06 eV. Although it is not sufficient to explain experimentally founded results, our result is close to DeGiorgi's experiment. We improve Gunnarsson's calculation by consid-

ering the electron-electron interaction and finite band-width effects explicitly. In conclusion, the unusual behavior of optical conductivity of the normal state A_3C_{60} reveals the fact that both the Coulomb interaction and electron-phonon interaction are important in examining dynamical properties of fullerene superconductors.

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Figure Captions

Figure 1. The optical conductivity as a function of ω for various electron-phonon coupling constants λ when $T = 0.001$ eV, $\varepsilon_F = 0.25$ eV and $1/\tau = 0.01$ eV. U is set to 0 for a reference. As λ is increased, the width and the weight of the Drude peak are reduced. The inset shows a decomposition of the total conductivity into the Drude and MIR parts for $\lambda = 0.7$.

Figure 2. The MIR spectra induced by the Coulomb interaction when $T = 0.001$ eV, $\varepsilon_F = 0.25$ eV, $1/\tau = 0.01$ eV and $\lambda = 0$. The inset shows a decomposition of the total conductivity, as in Fig. 1, into the Drude and MIR parts for $UN_F = 0.5$.

Figure 3. The MIR spectra as the impurity scattering rates $1/\tau$ are varied when $T = 0.005$ eV, $\varepsilon_F = 0.25$ eV, $\lambda = 0.7$ and $UN_F = 0.3$. When $1/\tau = 0.01$ eV the lower peak is mainly from the Coulomb interaction while the other peaks are from the electron-phonon interaction. As $1/\tau$ is increased, these peaks are merged altogether and finally evolve into a single broad peak around $0.06 \sim 0.1$ eV.

Figure 4. The total optical conductivity with it's Drude and MIR parts for $T = 0.005$ eV, $\varepsilon_F = 0.25$ eV, $1/\tau = 0.1$ eV, $\lambda = 0.7$ and $UN_F = 0.3$. The Drude part in the low frequency region is substantially suppressed due to the strong Coulomb interaction. Consequently, the missing spectral weight is transferred to the broad MIR peak, which peaks around 0.07 eV and extends well into the higher energy region. The ratio of the MIR spectral weight to the total intra-band spectral weight is 0.533.

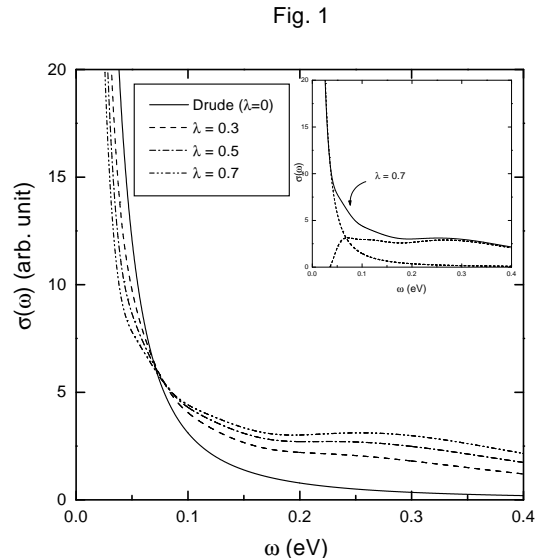


FIG. 1.

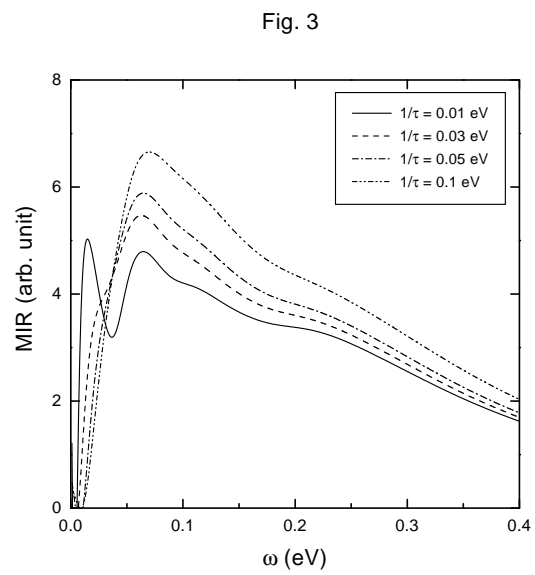
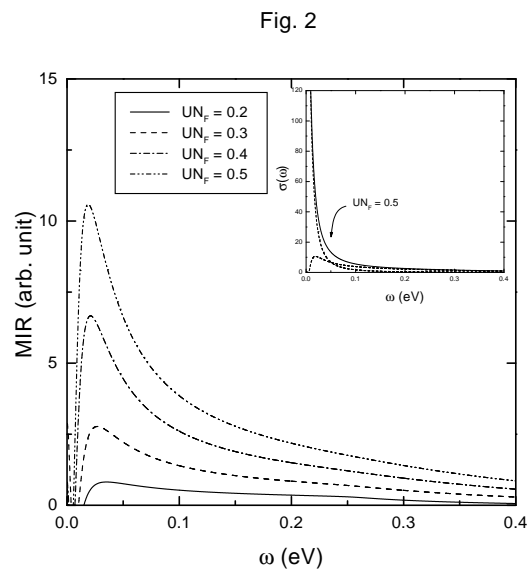


Fig. 4

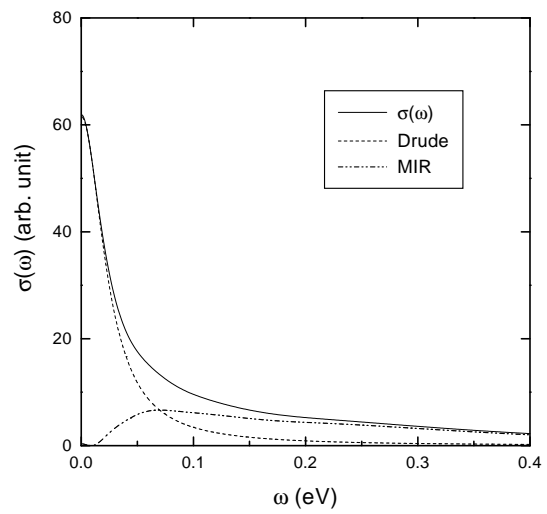


FIG. 4.